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THE TEMPERATURE DISTRIBUTION IN THE WALL OF A TUBE WITH NON-UNIFORM EXTERNAL HEATING AND INTERNAL COOLING

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Abstract—This paper presents a method of determining the temperature distribution in an internally cooled tube which is exposed to non-uniform radiation around its outer surface. Radial and circumferential heat transfer in the tube wall is considered and, at the tube inner surface, inter-radiation is accounted for in addition to convection. Also, the influence of tube spacing on the temperature distribution is included.

Equations are derived which are applicable when tubes are exposed to radiation from a flame which can be represented by an infinite plane source. In an appendix the equations are modified to apply to the case of a flame represented by a cylindrical source.

By use of finite-difference approximations and iteration, a method of solution to the steady state problem is outlined. This has been programmed for solution on a digital computer and the resulting isothermal curves arising from a given set of data are presented.

Also, an approximate solution is outlined which gives good agreement with the numerical solution when the difference in temperature between the radiating source and background is small.

NOMENCLATURE

A_{BM}, emissivity governing the radiant heat transfer from the background to the tube metal;

A_{FB}, emissivity governing the radiant heat transfer from the flame to the background;

A_{FM}, emissivity governing the radiant heat transfer from the flame to the tube metal;

A_{MM}, emissivity governing the radiant heat transfer between different parts of the tube inner surface;

D, width of each side of the square combustion chamber;

d, flame diameter;

dF_1, dF_2, elementary surface areas on the tube outer surface;

dF_i, dF_j, elementary surface areas on the tube inner surface;

dF_j', projected area of dF_j onto the surface of a unit hemisphere;

G, geometric factor—the ratio of flame perimeter to combustion chamber perimeter;

h, internal heat-transfer coefficient of the coolant;

I_2, intensity of normal radiation from dF_2;

I_j, intensity of normal radiation from dF_j;

N, number of tubes lining the combustion chamber;

q, heat flow;

r, radius; circular polar co-ordinate;

R_1, tube outer radius;

R_2, tube inner radius;

s, distance between the centres of adjacent tubes;

t, time;

T, tube absolute temperature at dF_1;

T_B, radiating absolute temperature of the background;

T_c, coolant absolute temperature;

T_F, radiating absolute temperature of the flame;

T_j $= T(R_2, \varphi)$, tube absolute temperature at dF_j;

$T(r, \theta)$, absolute temperature in the tube wall at the point defined by r, θ;

T^* $= \dfrac{T}{100}$;
$x, y, z,$ rectangular Cartesian co-ordinates.

Greek symbols
α, thermal diffusivity of the tube material;
α_1 $= \alpha_1(\theta)$, "plane" view angle of the flame at the point (R_1, θ);
α_3 $= \alpha_3(\theta)$, "plane" view angle of the background at the point (R_1, θ);
$\overline{\alpha_1(\theta)}$, "plane" view angle of the flame at the point (R_1, θ) for a cylindrical flame;
β, ratio of the tube outer diameter to the distance between centres of adjacent tubes;
γ, angle whose cosine is $4G/\pi$;
δ, angle whose cosine is β;
θ, circular and spherical polar co-ordinate; integration variable;
λ, thermal conductivity of the tube metal;
$\mu(\theta)$, "plane" view angle of the flame at the point (R_1, θ) for a cylindrical flame;
σ, Stefan–Boltzmann constant;
τ, angle between the line joining dF_1 to dF_2 and the outward drawn normal to dF_1; angle between the line joining dF_i to dF_j and the outward drawn normal to dF_i;
φ, spherical polar co-ordinate; integration variable;
∇^2, Laplacian operator in circular polar co-ordinates.

1. INTRODUCTION

In modern steam power plant an increasing proportion of steam superheat is supplied by radiant heat transfer. A knowledge of the thermal stresses within the walls of tubes used for this purpose is of great interest, particularly as temperature and heat flux move to more advanced conditions.

In order to calculate the thermal stresses, a knowledge of the prevailing temperature field is the first requirement. Most methods of calculating the temperature distribution within a tube wall given in the literature assume a known radiation heat transfer at the tube outer surface and no account is made of the view angle of a point on the outer surface (e.g. references [1] and [2]). Also, apart from Salzmann [1] who omits the effect of spacing between adjacent tubes, the effect of radiation at the tube inner surface is neglected and this can be considerable at high coolant temperatures.

This present study includes inter-radiation at the tube inner surface and takes account of the varying view angle at the outer surface. Two kinds of radiating source are considered. In the first case, equations are derived for an infinite plane radiating source and in the second these are modified to become applicable to a cylindrical source. The method is quite general in that it can be applied to tubes of different dimensions and also takes into account the effect of spacing between tubes.

A program incorporating the derived equations has been written for a digital computer and gives the steady-state temperature when tubes are heated by an infinite plane radiating source, representing a flame. The resulting temperatures from a typical set of data have been plotted graphically and the resulting isothermal lines shown.

It was suggested by a referee that mention be given to an approximate analytical solution to the problem and to this end Section 5 has been included. Results have been obtained from both numerical and analytical solutions and graphs are presented in Fig. 4 which compare the tube surface temperature profiles obtained from each method.

2. THEORETICAL CONSIDERATIONS AND ASSUMPTIONS

When a row of tubes is exposed to uniform radiation from a radiating source the heat flux around a tube outer surface will be non-uniform. The equations derived in the following are applicable to an infinite plane radiating source, as for example, can be assumed to be the case when a flame completely fills a combustion chamber (Fig. 1). However, with suitable modifications any type of radiating source can be accommodated; and Appendix 2 outlines the

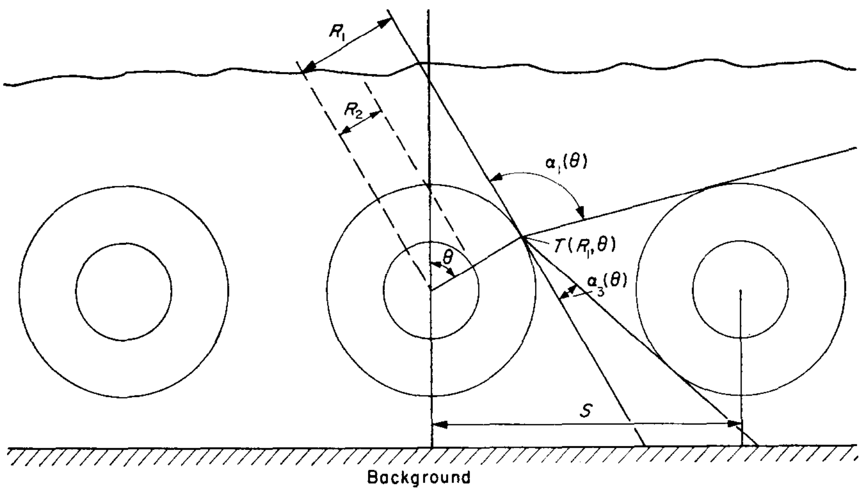

FIG. 1. Cross-section through three tubes showing notation used in analysis.

equations for a cylindrical source which can be assumed to be the case of a cylindrical flame enclosed in a square combustion chamber.

In the following, internally cooled tubes are considered which are subjected to external radiation whose intensity varies with angular co-ordinate only. Heat by direct radiation from a flame and diffused radiation from a background arrives at the tube outer surface and is conducted through the tube wall to the inner surface. At the inner surface the heat is dissipated by convection to a coolant. Also inter-radiation takes place between parts of the tube inner surface that are at different temperatures (Fig. 2). The effect of tube spacing is considered and the amount of diffused radiation decreases as the spacing between tubes decreases, and becomes zero when tubes touch each other.

The following assumptions are made:

(1) The convective heat transfer to the tube outer surface is small in comparison with the radiative heat transfer (probably less than 5 per cent) and therefore can be neglected.
(2) The flame radiates as an opaque Lambertian surface at uniform temperature.
(3) The background temperature is uniform.
(4) Heat flow along the tube length is small and can be neglected. Thus, conduction through the tube wall is confined to two dimensions.
(5) Radiation between the tube inner surface and the coolant can be neglected.
(6) The temperature distribution has an axis of symmetry, represented by the line through the tube centre and perpendicular to the line which joins the centres of adjacent tubes.
(7) The system is completely insulated.
(8) The background and surface of the tubes radiate as black bodies. The flame radiates as a grey body.

3. MATHEMATICAL INTERPRETATION OF THE PROBLEM

A point in the tube wall will be defined by the circular polar co-ordinates r, θ and the axis of symmetry defined by $\theta = 0$ and $\theta = \pi$.

3.1. *The heat transfer at the tube outer surface*

The tube outer surface receives radiant heat from the flame and from the background as shown in Fig. 2. By considering an elementary area on the tube outer surface and equating the radiant heat gained by the area to the heat conducted away from the area, the boundary condition at the tube outer surface is established.

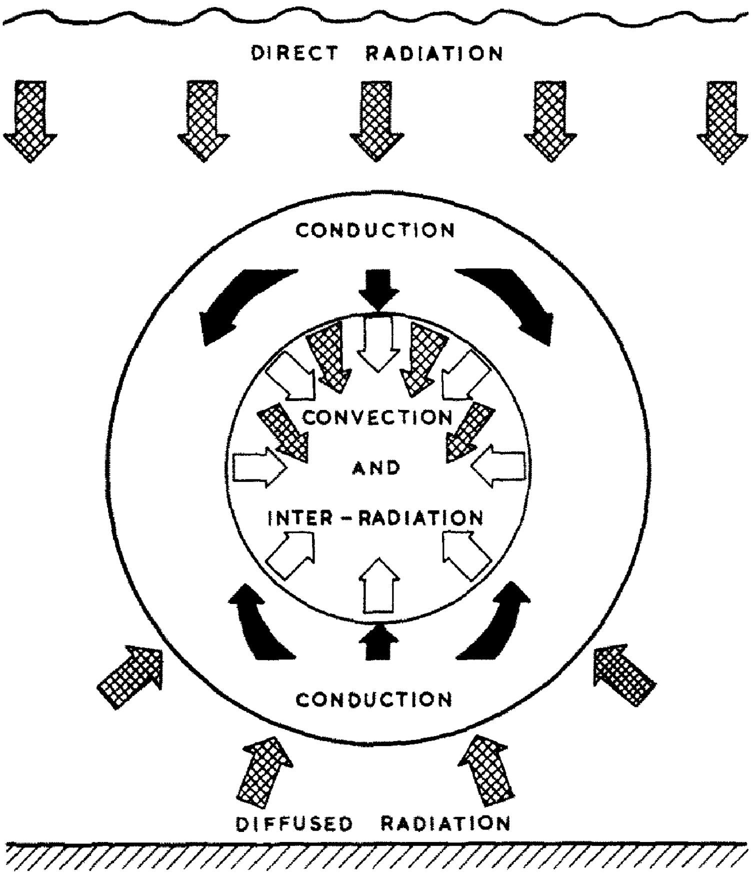

FIG. 2. Cross-section through a tube showing processes of heat transfer considered.

The condition, as derived in Appendix 1.1 becomes

$$\left.\begin{aligned} &0{\cdot}5\sigma\,\{A_{FM}\,[1-\cos\alpha_1(\theta)]\,[T_F^{*4}\\ &-T^{*4}(R_1,\theta)]+A_{BM}\,[1-\cos\alpha_3(\theta)]\\ &[T_B^{*4}-T^{*4}(R_1,\theta)]\}=\lambda\,\frac{\partial T(r,\theta)}{\partial r}\bigg|_{r=R_1}\end{aligned}\right\}\quad(1)$$

The first term on the left-hand side of equation (1) expresses the radiant heat flux between the flame and the tube outer surface, and the second term the radiant heat flux between the background and the tube outer surface. The right-hand side expresses conducted heat flux from the tube outer surface towards the tube inner surface.

3.2. *The heat transfer at the tube inner surface*

Heat arriving at the tube inner surface is led away by convection to the coolant. Also there is inter-radiation between those parts of the surface which are at different temperatures (Fig. 2). By establishing a heat balance for an elementary area on the tube inner surface the boundary condition at the tube inner surface is obtained. The heat received by the area due to conduction and the radiation from other parts of the surface is equated to the heat taken away by convection to the coolant and radiation from the area. As shown in Appendix 1.2 this reduces to

$$\left.\begin{aligned} & h[T(R_2, \theta) - T_c] + 0{\cdot}5\, A_{MM}\sigma \\ & \quad [2T^{*4}(R_2, \theta) - \int_0^\pi T^{*4}(R_2, \varphi) \\ & \quad \sin\varphi\, d\varphi] = \lambda \left.\frac{\partial T(r, \theta)}{\partial r}\right|_{r=R_2} \end{aligned}\right\} \quad (2)$$

On the left-hand side of equation (2) the first term is the heat flux to the coolant by convection and the second term is the inter-radiation between the parts of the tube surface which are at different temperatures. The right-hand side expresses heat flux arriving at the tube inner surface by conduction.

3.3. *Heat conduction through the tube wall*

For transient conditions, the temperature distribution in the tube wall is given by the solution to the equation of heat conduction in two dimensions [3]. This is expressed mathematically by

$$\nabla^2 T(r, \theta) = \frac{1}{\alpha}\frac{\partial T(r, \theta)}{\partial t} \quad (3)$$

Also the solution to equation (3) must satisfy the boundary conditions, i.e. equations (1) and (2), in which the flame and coolant temperature, and the emissivities can be (but are not necessarily) functions of time.

In the steady state the right-hand side of equation (3) is zero and the equation reduces to the Laplace equation in two dimensions. In circular polar co-ordinates, this is

$$\frac{\partial^2 T(r, \theta)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r, \theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T(r, \theta)}{\partial \theta^2} = 0 \quad (4)$$

The solution to this equation must also satisfy the boundary conditions and in this case the flame and coolant temperatures and the emissivities, in equations (1) and (2) are constants.

For both transient and steady-state conditions, the assumption that the temperature distribution is symmetric about the axis defined by $\theta = 0$ and $\theta = \pi$ is expressed by the relationship

$$T(r, \theta) = T(r, 2\pi - \theta) \quad (5)$$

In order to satisfy the boundary condition at the tube outer surface, i.e. equation (1), the background temperature must be known.

3.4. *Calculation of the background temperature*

The background temperature which depends upon the flame temperature will also depend upon the spacing between adjacent tubes. By performing a heat balance between the heat radiated from the flame to the background and the heat radiated from the background to the tube outer surface, the background temperature can be calculated. The heat balance, which is derived in Appendix 1.3 finally reduces to

$$\left.\begin{aligned} & (1 - \beta)A_{FB}\, [T_F^{*4} - T_B^{*4}] \\ & \quad = 0{\cdot}5\beta\, A_{BM} \int_0^\pi [1 - \cos \alpha_3(\theta)] \\ & \qquad [T_B^{*4} - T^{*4}(R_1, \theta)]\, d\theta \end{aligned}\right\} \quad (6)$$

for $0 < \beta \leqslant 1$.

The left-hand side of equation (6) expresses the radiant heat flux from the flame to the background, and the right-hand side the radiant heat flux from the background to the tube outer surface.

It should be noted that as β tends to unity, i.e. the case of tubes touching, so the left-hand side of equation (6), which is the radiant heat flux from the flame to the background, tends to zero.

4. METHOD OF SOLUTION

A numerical solution to the derived equations can be obtained by dividing the tube cross-section into a polar grid of discrete points and replacing the heat flux equation by finite-difference approximations to give a set of simultaneous algebraic equations which can be solved by iterative methods [4, 5]. One such method has been programmed for solution on the I.B.M. 7090 digital computer. This uses central finite-difference formulae, the two point formula for the first derivatives and the three-point formula for second derivatives. Due to the assumed symmetry of the problem only one half of the tube cross-section was considered.

Initially, values of temperature at the four corner points of the grid and a value for the background temperature are assumed; from these assumed values, values at each nodal point are estimated. This enables the integral appearing in the boundary equation at the tube inner surface, equation (2), to be evaluated numerically using Simpson's rule. By iteration new values of temperature are then obtained for all the nodal points of the mesh and, by using equation (6) a new value for the background temperature is obtained.

The cycle of iteration is repeated until convergence of the temperature values is obtained. Due to the non-linearity of the equations at the boundaries, it is possible for the temperature values to diverge and, in order to distinguish such a case, a test on the convergence has been included into the program.

The accuracy of the temperature values obtained is dependent upon the chosen number of nodal points in the grid. In general, the accuracy increases as the number of nodal points is increased.

Results from the program have been obtained and Fig. 3 shows the temperature isothermals resulting from a typical set of data.

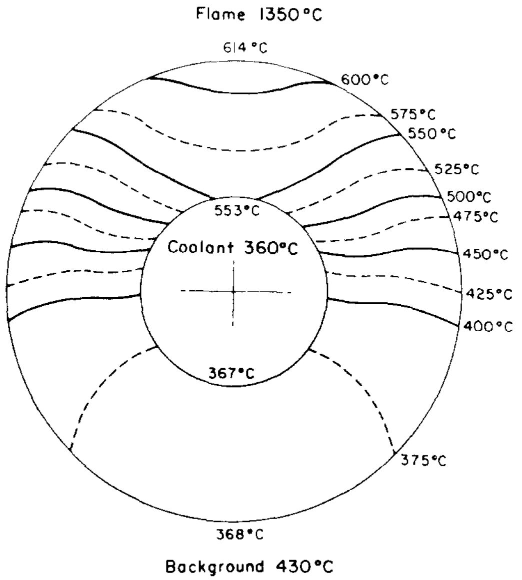

FIG. 3. Isothermal lines in the cross-section of a tube.

5. APPROXIMATE ANALYTICAL SOLUTION

An approximate solution can be obtained analytically by assuming that the temperature distributions on the tube boundaries can be represented as Fourier expansions. Due to the assumed symmetry of the problem only cosine terms need to be considered and by including only the first two terms in the Fourier series further approximation is obtained. Thus the outer and inner surface temperatures of the tube are represented respectively by

$$T(R_1, \theta) = A_0 + A_1 \cos \theta$$
$$T(R_2, \theta) = B_0 + B_1 \cos \theta$$

Within the tube wall the temperature is given by the solution to the partial differential equation (4), which can be obtained in terms of the above

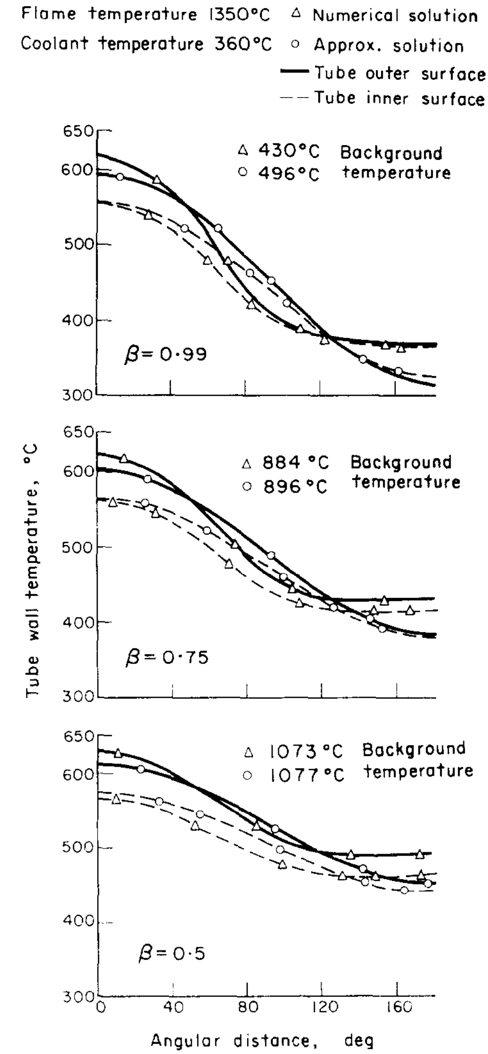

FIG. 4. Comparison of numerical and approximate solutions for tube surface temperature profiles.

constants by using the method of the separation of variables. Thus, when the values of the constants are known, the temperature at any point of the tube cross-section can be obtained.

To determine the constants A_0, A_1, B_0, B_1, four independent equations are required. Two such equations are derived by forming integrated heat flux balances on the inner and outer surfaces of the tube. At the inner surface the inter-radiation effect is included in the value of the heat-transfer coefficient, to ease solution. The additional equations are obtained by performing a local heat flux balance on both inner and outer surfaces of the tube at either of the positions $\theta = 0$ or $\theta = \pi$.

Using both the numerical and analytical solutions, tube temperature distributions have been evaluated for several sets of data and temperature profiles at the tube inner and outer surfaces are shown in Fig. 4. These results indicate that the approximate analytical solution will gain in accuracy as the temperature difference between the radiating source and background, which varies with the spacing between tubes, decreases.

6. CONCLUSIONS

Examination of results obtained from several sets of data shows the credibility of this method of temperature determination, and in view of this it is concluded that the method can be used as a basis for the derivation of the thermal stress distribution.

Comparison of the results from both numerical and approximate methods of solution shows that greater accuracy is obtained from the numerical method. Differences between the values obtained from each method decrease as the temperature between the source and background decreases.

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APPENDIX 1

Derivation of Formulae

1.1. *The boundary condition at the tube outer surface*

The flame radiates heat to the tube outer surface and also to the background where it is re-radiated to the tube outer surface. By considering an elementary area, denoted by dF_1, on the tube outer surface and equating the radiant heat input from the field to conduction through the tube wall, the condition to be satisfied at the tube outer surface is established.

Consider dF_1 at the centre of the base of a unit hemisphere, with the base lying in the tangent plane to the surface at dF_1 (Fig. 5). The radiant heat emitted from an elementary surface area of the flame which strikes dF_1 is equivalent to the radiant heat striking dF_1 from the central projection of the elementary surface area of the flame onto the surface of the hemisphere. Denote this projected surface area by dF_2. Then the radiant heat from dF_2 to dF_1 [3] is given by

$$d^2q_{21} = I_2 \, dF_2 \cos\frac{\pi}{2} \, dF_1 \cos\tau \tag{7}$$

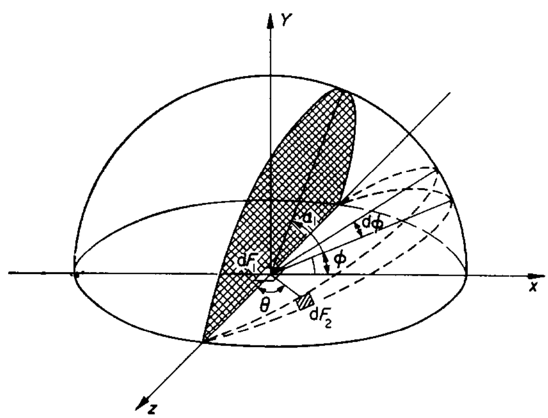

FIG. 5. Unit hemisphere indicating the determination of the "plane" view angle.

By summing the radiant heat emitted from all areas, such as dF_2, which are contained in that part of the surface of the hemisphere through which the flame is viewed by dF_1, the radiant heat from the flame to dF_1 is found.

Choose at dF_1 as origin a rectangular co-ordinate system with the x-axis lying in the base plane of the hemisphere, the y-axis lying along the outward drawn normal at dF_1 and the z-axis lying parallel to the tube axis (Fig. 5). Using spherical polar co-ordinates r, θ, φ with $r = 1$, the following relationships are satisfied:

$$dF_2 = \sin\theta \, d\theta \, d\varphi$$

$$\cos\tau = \sin\theta \sin\varphi$$

Substituting for dF_2 and $\cos\tau$ in equation (7), yields

$$d^2q_{21} = I_2 \, dF_1 \sin\varphi \, d\varphi \sin^2\theta \, d\theta \tag{8}$$

Thus the radiant heat emitted from the flame which strikes dF_1 is found by integrating equation (8) with respect to θ and φ, where θ extends over the range 0 to π and φ over the range 0 to α_1 (Fig. 5). α_1 is the value of φ through which the flame is viewed by dF_1 and it is termed the "plane" view angle of the flame at dF_1.

On integration and substitution of the limits, this becomes

$$dq_{21} = 0{\cdot}5 \, I_2 \, dF_1 \, \pi(1 - \cos\alpha_1)$$

From the Stefan–Boltzmann law [1]

$$I_2 = (\sigma/\pi) \, A_{FM} \, T_F^{*4}$$

Thus

$$dq_{21} = 0{\cdot}5\sigma \, A_{FM} \, T_F^{*4} \, (1 - \cos\alpha_1) \, dF_1$$

Similarly, the radiant heat from dF_1 to the flame, dq_{12}, is expressed as

$$dq_{12} = 0{\cdot}5\sigma \, A_{FM} \, T^{*4} \, (1 - \cos\alpha_1) \, dF_1$$

Thus the radiant heat flux into dF_1 from the flame, which is given by $(dq_{21} - dq_{12})/dF_1$ becomes

$$0{\cdot}5\sigma \, A_{FM} \, (1 - \cos\alpha_1) \, (T_F^{*4} - T^{*4})$$

Similarly the radiant heat flux into dF_1 from the background is given as

$$0{\cdot}5\sigma \, A_{BM} \, (1 - \cos\alpha_3) \, (T_B^{*4} - T^{*4}) \tag{9}$$

The values of α_1, α_3 and T will depend upon the position of dF_1. If circular polar co-ordinates r, θ are used to define a point in the tube wall, then α_1 and α_3 are both functions of θ, and T is a function of R_1 and θ (Fig. 1).

Thus the total heat flux input to dF_1, which is the sum of the heat flux due to the flame and that due to the background becomes

$$0{\cdot}5\sigma \, \{A_{FM} \, [1 - \cos\alpha_1(\theta)] \, [T_F^{*4} - T^{*4}(R_1, \theta)] + A_{BM} \, [1 - \cos\alpha_3(\theta)] \, [T_B^{*4} - T^{*4}(R_1, \theta)]\} \tag{10}$$

At the tube outer surface the heat flux conducted through the tube wall is given by

$$\lambda \frac{\partial T(r, \theta)}{\partial r}\Bigg|_{r=R_1} \tag{11}$$

Thus, equating expressions (10) and (11) yields the boundary condition to be satisfied at the tube outer surface as

$$0{\cdot}5\sigma \, \{A_{FM} \, [1 - \cos\alpha_1(\theta)] \, [T_F^{*4} - T^{*4}(R_1, \theta)] + A_{BM} \, [1 - \cos\alpha_3(\theta)] \, [T_B^{*4} - T^{*4}(R_1, \theta)]\} = \lambda \frac{\partial T(r, \theta)}{\partial r}\Bigg|_{r=R_1}$$

1.2. *The boundary condition at the tube inner surface*

At the inner surface heat is transferred to the coolant by convection and there is also inter-radiation heat transfer between those parts of the surface at different temperatures. The condition to be satisfied here is derived by considering an elementary area, denoted by dF_i, on the tube inner surface and equating the heat input due to conduction and radiation from the remaining area of tube inner surface to heat taken away by convection and radiation.

To calculate the heat input to dF_i by radiation from the remaining area of the tube inner surface, consider another elementary area on the inner surface not necessarily in the same plane, denoted by dF_j, and calculate the radiant heat emitted by dF_j which strikes dF_i (Fig. 6). By integrating dF_j over the inner surface area, the radiant heat input to dF_i is found.

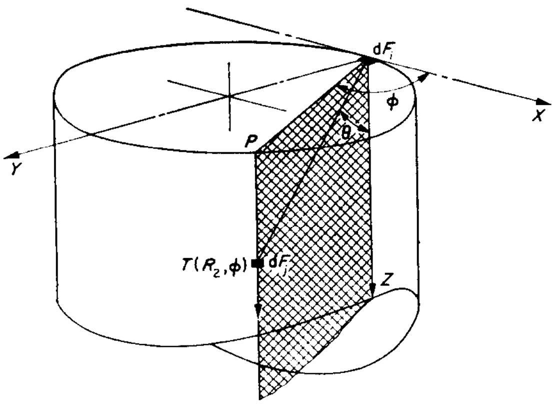

FIG. 6. Inner surface of a tube showing co-ordinate system.

Consider dF_i at the centre of the base of a unit hemisphere with the base lying in the tangent plane to the surface at dF_i. The radiant heat input to dF_i from dF_j is given by

$$d^2q_{ji} = I_j\, dF_j' \cos(\pi/2)\, dF_i \cos\tau \quad (12)$$

I_j is given in terms of the absolute temperature T_j of dF_j by the equation

$$I_j = (\sigma/\pi)\, A_{MM}\, T_j^{*4}$$

Expressions for dF_j' and τ are found by considering a rectangular Cartesian co-ordinate system at dF_i as origin in which the x-axis lies in the base plane of the hemisphere, the y-axis is the outward drawn normal to dF_i and the z-axis lies parallel to the tube axis (Fig. 6). Using spherical polar co-ordinates r, θ, φ with $r = 1$, gives

$$dF_j' = \sin\theta\, d\theta\, d\varphi$$

$$\cos\tau = \sin\theta \sin\varphi$$

Also the total radiant heat input to dF_i is found by integrating equation (12) over the surface area of the hemisphere. Substituting for I_j, dF_j' and $\cos\tau$ this becomes

$$dq_{ji} = A_{MM}\, dF_i \int_0^\pi T_j^{*4} \sin\varphi\, d\varphi \int_0^\pi \sin^2\theta\, d\theta$$

T_j^{*4} is under the integral sign since its value is a function of φ, and, of course R_2. Thus replacing T_j by $T(R_2, \varphi)$ and performing the integration with respect to θ yields an expression similar to that given by Saltzmann [1].

This is

$$dq_{ji} = 0{\cdot}5\sigma\, A_{MM}\, dF_i \int_0^\pi T_j^{*4}(R_2, \varphi) \sin\varphi\, d\varphi \quad (13)$$

Using the Stefan–Boltzmann law and noting that the temperature of dF_i is a function of the circular polar co-ordinates r and θ, the radiant heat flux from dF_i is given as

$$(\sigma/\pi)\, A_{MM}\, T^{*4}(R_2, \theta) \quad (14)$$

Also, if the coolant temperature is assumed constant, the heat flux transferred from dF_i by convection is expressed as

$$h[T(R_2, \theta) - T_c] \quad (15)$$

Finally, there is the heat flux received by dF_i by conduction, this is given as

$$\lambda \left.\frac{\partial T(r, \theta)}{\partial r}\right|_{r=R_2} \quad (16)$$

Using expressions (13)–(16) and equating radiant heat flux input to radiant heat flux output yields the condition to be satisfied at the inner surface. This becomes

$$h[T(R_2, \theta) - T_c] + 0{\cdot}5\sigma\, A_{MM}[2T^{*4}(R_2, \theta) - \int_0^\pi T^{*4}(R_2, \varphi) \sin\varphi\, d\varphi] = \lambda \left.\frac{\partial T(r, \theta)}{\partial r}\right|_{r=R_2}$$

1.3. *The equation to determine the background temperature*

The background temperature is found by equating the heat input to the background due to radiation from the flame, to the heat output due to radiation between background and tubes. Since uniform flame and background temperatures are assumed, only one tube need be considered (Fig. 1). If s denotes the distance between the centres of adjacent tubes, then for unit length of flame in the direction of the tube axis, the radiation between flame and background is, by the Stefan–Boltzmann law,

$$(s - 2R_1)\, \sigma A_{FB}\, [T_F^{*4} - T_B^{*4}] \quad (17)$$

Now expression (9) gives the heat flux from the background to an elementary area on the tube outer surface. This is

$$0{\cdot}5\sigma\, A_{BM}(1 - \cos\alpha_3)\, (T_B^{*4} - T^{*4})$$

For unit length of tube, an elementary area on the tube outer surface is $R_1\,d\theta$. Remembering that α_3 and T depend on θ, then the radiation into the elementary area becomes

$$0{\cdot}5\sigma A_{BM}[1-\cos\alpha_3(\theta)][T_B^{*4}-T^{*4}(R_1,\theta)]\,R_1\,d\theta$$

Thus the total radiation between the background and the tube outer surface is

$$0{\cdot}5\sigma A_{BM}R_1\int_0^{2\pi}[1-\cos\alpha_3(\theta)][T_B^{*4}-T^{*4}(R_1,\theta)]\,d\theta \quad (18)$$

Equating this expression with (17), yields

$$(s-2R_1)A_{FB}[T_F^{*4}-T_B^{*4}] = 0{\cdot}5\,R_1\,A_{BM} \times\int_0^{2\pi}[1-\cos\alpha_3(\theta)][T_B^{*4}-T^{*4}(R_1,\theta)]\,d\theta$$

Dividing each side by s gives, since $\beta = 2\,R_1/s$,

$$(1-\beta)A_{FB}[T_F^{*4}-T_B^{*4}] = 0{\cdot}25\beta\,A_{BM} \times\int_0^{2\pi}[1-\cos\alpha_3(\theta)][T_B^{*4}-T^{*4}(R_1,\theta)]\,d\theta \quad (19)$$

From the expression for $1-\cos\alpha_3(\theta)$, given in Appendix 1.4 it can be seen that

$$\int_0^{2\pi}[1-\cos\alpha_3(\theta)][T_B^{*4}-T^{*4}(R_1,\theta)]\,d\theta = 2\int_0^{\pi}[1-\cos\alpha_3(\theta)][T_B^{*4}-T^{*4}(R_1,\theta)]\,d\theta$$

Thus expression (19) finally becomes

$$(1-\beta)A_{FB}[T_F^{*4}-T_B^{*4}] = 0{\cdot}5\beta\,A_{BM} \times\int_0^{\pi}[1-\cos\alpha_3(\theta)][T_B^{*4}-T^{4*}(R_1,\theta)]\,d\theta$$

1.4. *Formulae for* $1-\cos\alpha_1(\theta)$ *and* $1-\cos\alpha_3(\theta)$

Values for $1-\cos\alpha_1(\theta)$ and $1-\cos\alpha_3(\theta)$ will depend upon the value of θ. By considering a point (R_1, θ) on the tube outer surface and using known trigonometric relationships, the following expressions for $1-\cos\alpha_1(\theta)$ and $1-\cos\alpha_3(\theta)$ are derived:

$$1-\cos\alpha_1(\theta) = 1 + \frac{\dfrac{4\cos\theta}{\beta}\left\{\dfrac{1}{\beta^2}-\dfrac{\sin\theta}{\beta}\right\}^{\frac{1}{2}} + \left\{1-\dfrac{2\sin\theta}{\beta}\right\}}{1+\dfrac{4}{\beta^2}-\dfrac{4}{\beta}\sin\theta}$$

$$\text{for } 0 \leqslant \theta \leqslant (\pi/2)+\delta$$

$$= 0 \qquad \text{for } (\pi/2)+\delta < \theta \leqslant \pi$$

$$1-\cos\alpha_3(\theta) = 1-\cos\alpha_1(\pi-\theta) \qquad \text{for } 0 \leqslant \theta \leqslant \pi$$

where $\delta = \cos^{-1}\beta$.

APPENDIX 2

Modifications to formulae for a cylindrical flame enclosed by a square combustion chamber

The equations derived in Appendix 1 are applicable, as stated in Section 2 to an infinite plane radiating source representing the flame. This is the case, for instance, when a flame completely fills a combustion chamber. However, it is possible to include other flame shapes by considering the geometrical configuration of the system and appropriately modifying the derived equations.

Consider for example the case of a cylindrical flame situated at the centre of a square combustion chamber, the flame being represented by a cylindrical radiating source. Here the boundary condition at the tube outer surface and the background temperature are affected by the geometry of the system.

Considering the tube situated at the middle of one side of the combustion chamber (so as to retain the condition of symmetry), the modifications are as follows:

2.1. *The boundary condition at the tube outer surface*

The basic form of the equation to be satisfied at the tube outer surface remains the same but the formulae which determine $1-\cos\alpha_1(\theta)$ are altered. Let the new expression for $1-\cos\alpha_1(\theta)$ be denoted by $1-\cos\overline{\alpha_1(\theta)}$. By considering a point (R_1, θ) on the tube outer surface, it can be shown that for $0 \leqslant \theta \leqslant \gamma$

$$1-\cos\overline{\alpha_1(\theta)} = (32G^2/\pi^2)$$

G is called the geometric factor and is defined as the ratio of the flame perimeter to the combustion chamber perimeter. γ is given by the equation

$$\gamma = \cos^{-1}\{4G/\pi\}$$

and is the limiting value of θ beyond which the amount of flame viewed from the point (R_1, θ) decreases as θ increases.

For $\gamma < \theta \leqslant \pi$ there are two possibilities, either (a)

$$1 - \cos\mu(\theta) \leqslant 1 - \cos\alpha_1(\theta)$$

or (b)

$$\begin{aligned} 1 - \cos\mu(\theta) &< 1 - \cos\alpha_1(\theta) && \text{for } \gamma < \theta < \theta_1 \\ &= 1 - \cos\alpha_1(\theta) && \text{for } \theta = \theta_1 \\ &\geqslant 1 - \cos\alpha_1(\theta) && \text{for } \theta_1 < \theta \leqslant \pi \end{aligned}$$

Where $\mu(\theta)$ denotes the "plane" view angle of the flame at the point (R_1, θ) and is given by the equation

$$1 - \cos\mu(\theta) = 1 - \{1 - (16G^2/\pi^2\}^{\frac{1}{2}} \sin\theta + (4G/\pi)\cos\theta$$

If (a) then

$$\begin{aligned} 1 - \cos\overline{\alpha_1(\theta)} &= 1 - \cos\mu(\theta) && \text{for } \gamma < \theta < \pi - \gamma \\ &= 0 && \text{for } \pi - \gamma \leqslant \theta \leqslant \pi \end{aligned}$$

If (b) then

$$\begin{aligned} 1 - \cos\overline{\alpha_1(\theta)} &= 1 - \cos\mu(\theta) && \text{for } \gamma \leqslant \theta < \theta_1 \\ &= 1 - \cos\alpha_1(\theta) && \text{for } \theta_1 \leqslant \theta < (\pi/2) + \delta \\ &= 0 && \text{for } (\pi/2) + \delta \leqslant \theta \leqslant \pi \end{aligned}$$

In the formulation of the above expressions it has been assumed that the tube diameter is very small in comparison to the width of the chamber sides, so that the ratio of tube diameter to the width of the chamber side is taken to be zero.

The expression to be satisfied at the tube outer surface becomes

$$0{\cdot}5\sigma \left\{ A_{FM}[1 - \cos\overline{\alpha_1(\theta)}][T_F{}^{*4} - T^{*4}(R_1, \theta)] + A_{BM}[1 - \cos\alpha_3(\theta)][T_B^{*4} - T^{*4}(R_1, \theta)] \right\} = \lambda \left. \frac{\partial T(r, \theta)}{\partial r} \right|_{r=R_1} \quad (20)$$

2.2. *Calculation of the background temperature*

The background temperature is found, as before, by equating the radiative heat input to the radiative heat output at the background. However, in this case, since flame and combustion chamber perimeters differ, total radiation must be considered.

Let D denote the width of each side of the combustion chamber, d the flame diameter, and N the number of tubes lining the combustion chamber, then the flame radiation/unit area of background is given as

$$\frac{(s - 2R_1)N}{4D} \frac{\pi d}{4D} \sigma A_{FB} T_F{}^{*4}$$

The radiation/unit area from the background is

$$\frac{(s - 2R_1)N}{4D} \sigma A_{FB} T_B^{*4}$$

Thus, the net radiation/unit area input to the background becomes

$$\frac{(s - 2R_1)N}{4D} \sigma A_{FB} \left[\frac{\pi d}{4D} T_F^{*4} - T_B^{*4} \right] \quad (21)$$

Using the expression (18), the radiation/unit area exchanged between the background and the outer surface of the tubes becomes

$$0{\cdot}5\,(N/4D)\,\sigma A_{BM} R_1 \int_0^{2\pi} [1 - \cos\alpha_3(\theta)] [T_B^{*4} - T^{*4}(R_1, \theta)]\, d\theta \quad (22)$$

Equating expressions (21) and (22), and simplifying gives

$$(s - 2R_1) A_{FB} [(\pi d/4D) T_F^{*4} - T_B^{*4}] = R_1 A_{BM} \int_0^{\pi} [1 - \cos\alpha_3(\theta)] [T_B^{*4} - T^{*4}(R_1, \theta)]\, d\theta$$

Dividing both sides by s and, since G is the ratio of flame perimeter to the perimeter of the combustion chamber, substituting G for $\pi d/4D$ gives the expression for the calculation of the background temperature. This is

$$(1 - \beta) A_{FB} [G T_F^{*4} - T_B^{*4}] = 0{\cdot}5\beta A_{BM} \int_0^{\pi} [1 - \cos\alpha_3(\theta)] [T_B^{*4} - T^{*4}(R_1, \theta)]\, d\theta \quad (23)$$

Thus, the temperature distribution prevailing in the cross-section of the tubes which are situated at the mid-point of the sides of the combustion chamber is given by solving equations (20) and (23) together with equations (2), (4) and (5). To obtain the temperature distribution for the tubes which are situated in the corners of the combustion chamber, substitute $G\sqrt{2}$ for G in equation (23) and in the expressions for $1 - \cos \overline{\alpha_1(\theta)}$ and γ and solve together with equations (2), (4), (5) and (20).

Résumé—On expose une méthode de détermination de la distribution de température dans un tube refroidi intérieurement et exposé à un rayonnement non uniforme autour de sa surface extérieure. La transport de chaleur radial et circonférentiel dans la paroi du tube est étudié et l'on tient compte du rayonnement de la surface intérieure du tube en plus de la convection. On a ajouté également l'influence de l'espacement des tubes sur la distribution de température.

On a écrit les équations applicables au cas de tubes exposés au rayonnement d'une flamme qui peut être assimilée à une source plane infinie. En annexe, les équations sont modifiées pour s'appliquer au cas d'une flamme assimilée à une source cylindrique.

En employant des approximations de différences finies et une itération, une méthode de résolution en régime permanent a été ébauchée. Elle a été programmée sur un calculateur numérique et l'on a présenté les isothermes correspondant à un ensemble donnée de paramètres.

On donne également un aperçu d'une solution approchée qui est en bon accord avec la solution numérique lorsque la différence de température entre la source de rayonnement et l'environnement est faible.

Zusammenfassung—Zur Bestimmung der Temperaturverteilung in einem innen gekülhten Rohr dessen äusserer Umfang einer ungleichmässigen Strahlung unterworfen ist wird eine Methode angegeben. Der Wärmetransport in der Rohrwand in Radial- und in Umfangsrichtung ist berücksichtigt und an der inneren Rohroberfläche wird zur Konvektion ein Strahlungsanteil zugeschlagen. Auch ist der Einfluss des Raumes zwischen den Rohren auf die Temperaturverteilung in die Betrachtung einbezogen.

Die abgeleiteten Gleichungen gelten für Rohre welche einer Flammstrahlung ausgesetzt sind die von einer unendlichen ebenen Quelle zu kommen scheint. Im Anhang sind die Gleichungen für die von einer zylindrischen Quelle herrührenden Strahlung modifiziert.

Mit Hilfe der Näherungen durch endliche Differenzen und Iteration erhalt man eine Lösungsmethode für den stationären Fall. Die danach auf einer digitalen Rechenmaschine für eine Reihe gegebener Werte ermittelten Isothermenkurven sind angegeben. Auch eine Näherungslösung ist wiedergegeben die für kleine Temperaturdifferenzen zwischen strahlender Quelle und Hintergrund zu guter Übereinstimmung mit der numerischen Lösung führt.

Аннотация—В статье предложен метод определения распределения температуры в охлаждаемой изнутри трубе, наружная поверхность которой подвергается действию неоднородного излучения. Рассматривается радиальная и угловая теплопроводность в стенке трубы и на внутренней ее поверхности. Помимо конвекции учитывается взаимное излучение. Учитывается также влияние расположения трубы на распределение температуры.

Выведены уравнения, применимые к случаю воздействия на трубы излучения пламени, которое можно представить в виде бесконечного плоского источника. В приложении уравнения модифицированы применительно к случаю пламени, представленного в виде цилиндрического источника.

С помощью аппроксимации конечными разностями и итерации разработан метод решения стационарной задачи. Задача запрограммирована для решения на счетной машине. На основе полученных данных представлены результирующие изотермические кривые.

Выведено приближенное решение, дающее хорошее согласие с численным решением при небольшой разности температур между источником излучения и фоном.